Transformations, Symmetry, and the Starblanket
The Mathematics of Indigenous Design

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Acknowledgements

Geometric Transformations
Symmetry and Groups
Starblankets
Sequences and Series

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Transformations, Symmetry, and the Starblanket
1 Acknowledgements

2 Geometric Transformations

- Unified Canadian Aboriginal Syllabics
- Birch Bark Biting
1  Acknowledgements

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3  Symmetry and Groups
   - West Coast Art and Totem Poles
   - Quill Boxes
   - Pueblo Pottery
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4 Starblankets
- Some Shapes
- Starblanket Designs
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4. Starblankets
   - Some Shapes
   - Starblanket Designs

5. Sequences and Series
   - The Square Perimeter Sequence
   - The Starblanket Perimeter Sequence
   - The Square Area Sequence
   - The Starblanket Area Sequence
   - Arithmetic Series
Acknowledgements
I would like to thank the Banff International Research Station for supporting this work.

I would like to thank Elders Betty McKenna and the late Narcisse Blood for their help.
Acknowledgements
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Starblankets
Sequences and Series

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Betty McKenna
Geometric Transformations
Geometric Transformations

- Translation
- Rotation
- Reflection
- Glide-reflection

Key:
- Translation axis
- Two-fold rotation
- Reflection axis
- Glide-reflection axis
Unified Canadian Aboriginal Syllabics
## Acknowledgements

Geometric Transformations  
Symmetry and Groups  
Starblankets  
Sequences and Series

# Unified Canadian Aboriginal Syllabics

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Birch Bark Biting
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Symmetry and Groups
Symmetry Groups

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West Coast Art and Totem Poles
Art Examples: Drawings
Art Examples: Drawings
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Quill Boxes
Pueblo Pottery

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Pueblo Pottery
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Transformations, Symmetry, and the Starblanket
Starblankets
Some Shapes
Some Shapes

Square

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Some Shapes

Square

Rectangle
Some Shapes

Square

Rectangle

Rhombus
Some Shapes

- Square
- Rectangle
- Rhombus
- Parallelogram

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Starblanket Designs
Starblanket

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Transformations, Symmetry, and the Starblanket
Another Starblanket
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Transformations, Symmetry, and the Starblanket
Non-Rhombus Starblanket
Seven-pointed Starblanket
Sequences and Series
The Square Perimeter Sequence
The Square Perimeter Sequence: Radius 1, Length 4
The Square Perimeter Sequence: Radius 2, Length 8
The Square Perimeter Sequence: Radius 3, Length 12
The Square Perimeter Sequence: Radius 4, Length 16
The Square Perimeter Sequence: Radius 5, Length 20
Finite Differences for Square Perimeter Sequence

How can we continue the square perimeter sequence 4, 8, 12, 16, 20 without using algebra?
How can we continue the square perimeter sequence 4, 8, 12, 16, 20 without using algebra? We can use a finite difference scheme.

\begin{align*}
4 & \\
8 & \\
12 & \\
16 & \\
20 & \\
\end{align*}
Finite Differences for Square Perimeter Sequence

How can we continue the square perimeter sequence 4, 8, 12, 16, 20 without using algebra? We can use a finite difference scheme.

4 8 12 16 20
4 4 4 4
4
Finite Differences for Square Perimeter Sequence

How can we continue the square perimeter sequence 4, 8, 12, 16, 20 without using algebra? We can use a finite difference scheme.

Notice we have constant first differences, also known as a **common difference**. Can we use this table to predict the next square perimeter number?
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Notice we have constant first differences, also known as a common difference. Can we use this table to predict the next square perimeter number?
The Starblanket Perimeter Sequence
The Starblanket Perimeter Sequence: Radius 1, Length 16
The Starblanket Perimeter Sequence: Radius 2, Length 32
The Starblanket Perimeter Sequence: Radius 3, Length 48
The Starblanket Perimeter Sequence: Radius 4, Length 64
The Starblanket Perimeter Sequence: Radius 5, Length 80
Finite Differences for Starblanket Perimeter Sequence

How can we continue the starblanket perimeter sequence 16, 32, 48, 64, 80 without using algebra?

We can use a finite difference scheme. Notice we have constant first differences, like other 1D figurate numbers (perimeters of squares, perimeters of triangles). Can we use this table to predict the next starblanket perimeter number?
Finite Differences for Starblanket Perimeter Sequence

How can we continue the starblanket perimeter sequence 16, 32, 48, 64, 80 without using algebra? We can use a finite difference scheme.

16
32
48
64
80
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The Square Area Sequence
The Square Area Sequence: Radius 1, Area 1
The Square Area Sequence: Radius 2, Area 4
The Square Area Sequence: Radius 3, Area 9
The Square Area Sequence: Radius 4, Area 16
The Square Area Sequence: Radius 5, Area 25
Finite Differences for Square Area Sequence

How can we continue the square area sequence 1, 4, 9, 16, 25 without using algebra? We can again use a finite difference scheme.
Finite Differences for Square Area Sequence

How can we continue the square area sequence 1, 4, 9, 16, 25 without using algebra? We can again use a finite difference scheme.

\[
\begin{array}{c}
1 \\
4 \\
9 \\
16 \\
25 \\
\end{array}
\]

Notice we have constant second differences. Can we use this table to predict the next square area number?
Finite Differences for Square Area Sequence

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```
1
4
9
16
25
```

```
3
5
7
9
```

```
2
2
2
```

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The Starblanket Area Sequence
The Starblanket Area Sequence: Radius 1, Patches 8
The Starblanket Area Sequence: Radius 2, Patches 32
The Starblanket Area Sequence: Radius 3, Patches 72
The Starblanket Area Sequence: Radius 4, Patches 128
The Starblanket Area Sequence: Radius 5, Patches 200
Finite Differences for Starblanket Area Sequence

How can we continue the starblanket area sequence 8, 32, 72, 128, 200 without using algebra? We can again use a finite difference scheme.
Finite Differences for Starblanket Area Sequence

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<table>
<thead>
<tr>
<th></th>
<th>8</th>
<th>32</th>
<th>72</th>
<th>128</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ</td>
<td></td>
<td>24</td>
<td>40</td>
<td>56</td>
<td>88</td>
</tr>
<tr>
<td>Δ</td>
<td></td>
<td>16</td>
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Arithmetic Series
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First Nations University: How Many Patches?
First Nations University: How Many Patches?

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Transformations, Symmetry, and the Starblanket
First Nations University: How Many Patches?
First Nations University: How Many Patches?

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Thank You for Your Interest